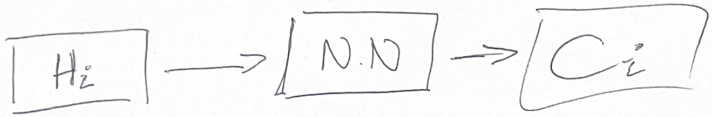
At t_i



$\Rightarrow$ ~~Minimize~~ Minimize the lower bound of $I(x_{i+1}, C_i)$

*CPC paper

I.B.I

- There exists a structured dist: $p(H_i, X_{i+1})$
- Relevant info in $H_i \equiv I(H_i; X_{i+1})$
- $\hat{H}_i$ = minimal sufficient statistics,
  = simplest mapping of H that captures $I(H_i, X_{i+1})$
- Best representation found by: minimizing

$$\mathcal{L}[p(\hat{H}|H)] = \underbrace{I(H;\hat{H})}_{\text{compressive}} - \beta \underbrace{(I(\hat{H}; X_{i+1}))}_{\text{"predictive info preserving"}}$$

- Data processing inequality:

$$I(H, X_{i+1}) \geq I(\hat{H}, X_{i+1}) \geq I(\hat{X}_{i+1}, X_{i+1})$$

- Rate distortion theory:
  How much can you compress input & maintain a distortion less/eq. D on reconstruction?

- Distortion Measure

$$d(H, \hat{H}) = KL[p(x_{i+1}|H_i) \| p(x_{i+1}|\hat{H})]$$

- ~~When $\hat{H}$ optimized sufficient statistic~~

$$\mathbb{E}[d(H, \hat{H})] = I(H; x_{i+1}|\hat{H})$$

↑ Residual info in history about x_{i+1} not captured by $\hat{H}$.

↑ Want to reduce this value.

- Rewrite the loss:

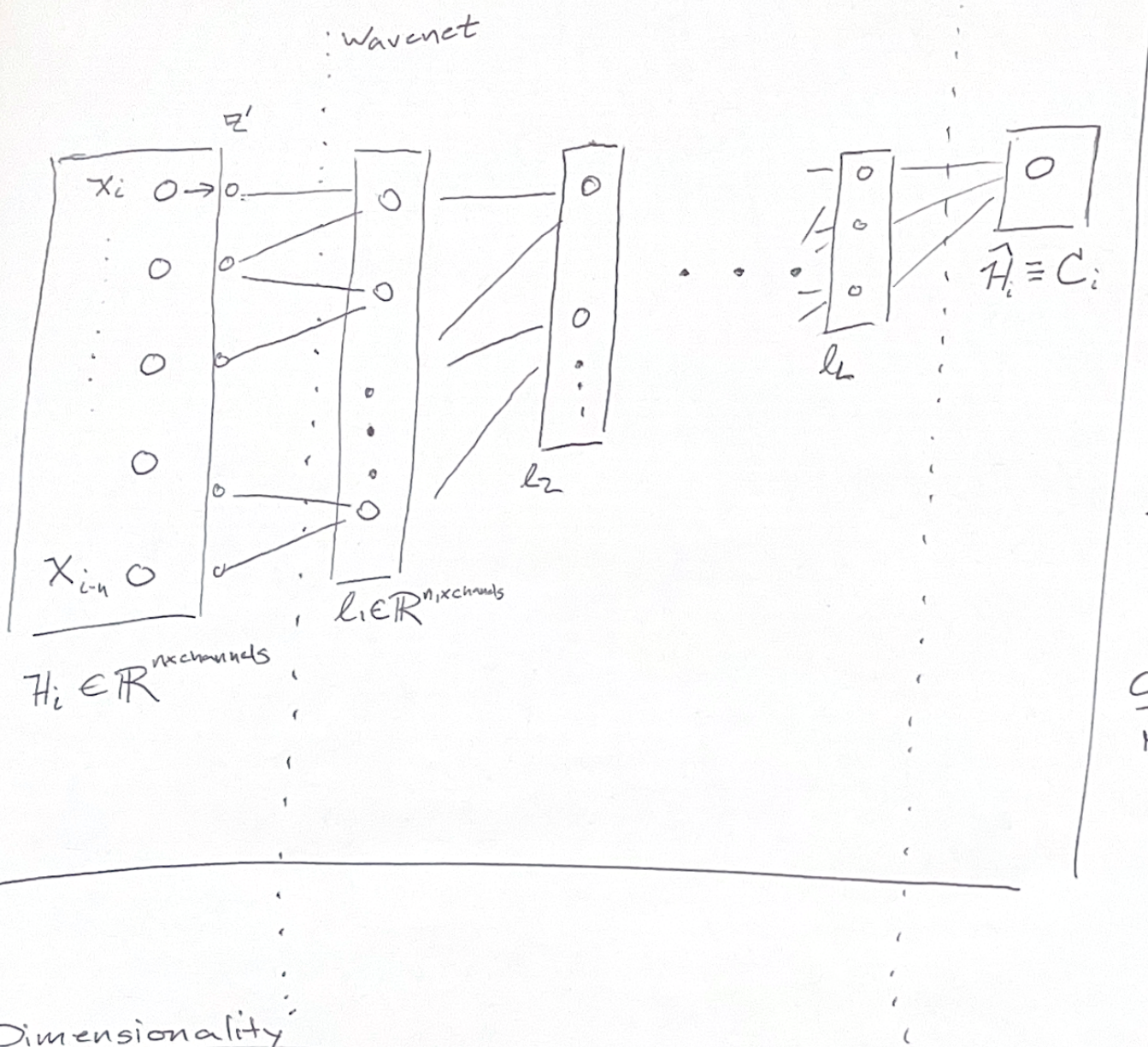
$$\mathcal{L} = \underbrace{I(H; \hat{H})}_{\text{compression}} + \underbrace{\beta\left(I(H; x_{i+1}|\hat{H})\right)}_{\text{Residual Information.}}$$

- IB Distortion

$$D_N = I(H_i; x_{i+1}|\hat{x}_{i+1})$$

- Representational Complexity of Output

$$R_N = I(H_i; \hat{x}_{i+1})$$



IB Goals:

1. minimize $I(H_i; C_i)$
2. maximize $I(C_i; X_{i+1})$ *
   OR
   minimize $I(H_i; X_{i+1} | C_i)$

CPC Training:

Maximize LB of $I(X_{i+1}; C_i)$*

Dimensionality

$H_i = \mathbb{R}^{n \times \text{channels}}$

$z' = \mathbb{R}^{n \times \text{latent_dim}}$

$l_i = \mathbb{R}^{(\sum_{j=0}^{i} (\text{dilation}_j, \text{kernel}_j), \text{features}_i)}$

$c_i = \mathbb{R}^{\text{context_dim}}$